

Unintegrated Double Parton Distributions - a Summary

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Abstract. We present main elements of the construction of unintegrated double parton distribution functions which depend on transverse momenta of partons. We follow the method proposed by Kimber, Martin and Ryskin for a construction of unintegrated single parton distributions from the standard parton distribution functions.

INTRODUCTION

Double parton scattering belongs to a class of multi-parton interactions in hadronic scattering in which two systems with hard scales are produced in one event due to two independent parton-parton interactions, see [1] for a theoretical introduction. Such events were observed by both the Fermilab [2, 3, 4] and CERN experiments [5, 6, 7]. The standard QCD description of such processes is based on double parton distribution functions (DPDFs) and collinear factorization of cross sections, which is a generalization of the single parton scattering description with the well known single parton distribution functions (PDFs). We present a first attempt to a further generalization in which the DPDFs start to depend on transverse momenta of active partons, thus may be used in k_\perp -factorized cross sections with off-shell partons initiating hard scatterings. We follow the method proposed in References [8, 9] for the construction of unintegrated PDFs by unfolding the last step in the QCD evolution of PDFs. This is why we call our distributions unintegrated DPDFs (UDPDFs). The full documentation of our construction can be found in Reference [10].

DOUBLE PARTON DISTRIBUTIONS

The DPDFs, $D_{a_1 a_2}(x_1, x_2, Q_1, Q_2)$, depend on positive, longitudinal momentum fractions, $x_{1,2}$, of two partons of flavors (including gluon) $a_{1,2}$, and also on two hard scales $Q_{1,2}$. To save on space we switch to their double Mellin moments

$$\tilde{D}_{a_1 a_2}(n_1, n_2, Q_1, Q_2) = \int_0^1 dx_1 \int_0^1 dx_2 x_1^{n_1} x_2^{n_2} \theta(1 - x_1 - x_2) D_{a_1 a_2}(x_1, x_2, Q_1, Q_2). \quad (1)$$

In the leading logarithmic approximation, the DPDFs evolve with hard scales according to the equation [11, 12]

$$\begin{aligned} \tilde{D}_{a_1 a_2}(n_1, n_2, Q_1, Q_2) &= \sum_{a', a''} \left\{ \tilde{E}_{a_1 a'}(n_1, Q_1, Q_0) \tilde{E}_{a_2 a''}(n_2, Q_2, Q_0) \tilde{D}_{a' a''}(n_1, n_2, Q_0, Q_0) \right. \\ &\quad \left. + \int_{Q_0^2}^{Q_{\min}^2} \frac{dQ_s^2}{Q_s^2} \tilde{E}_{a_1 a'}(n_1, Q_1, Q_s) \tilde{E}_{a_2 a''}(n_2, Q_2, Q_s) \tilde{D}_{a' a''}^{(sp)}(n_1, n_2, Q_s) \right\}, \end{aligned} \quad (2)$$

where $Q_{\min}^2 = \min\{Q_1^2, Q_2^2\}$, $\tilde{D}_{a' a''}(n_1, n_2, Q_0, Q_0)$ is an initial condition and

$$\tilde{D}_{a' a''}^{(sp)}(n_1, n_2, Q_s) = \frac{\alpha_s(Q_s)}{2\pi} \sum_a \tilde{D}_a(n_1 + n_2, Q_s) \int_0^1 dz z^{n_1} (1 - z)^{n_2} P_{a \rightarrow a' a''}(z). \quad (3)$$

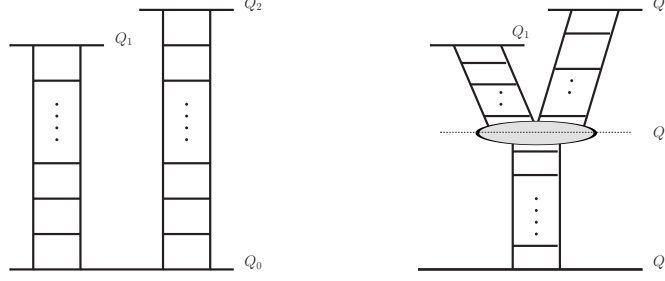


FIGURE 1. Schematic illustration of two contributions to the evolution of DPDFs, see Equation 2.

In Equation 3 $P_{a \rightarrow a' a''}(z) \equiv P_{a' a}(z)$ is the leading order Altarelli-Parisi splitting function and $\tilde{D}_a(n_1 + n_2, Q_s)$ is the single PDF. The structure of Equation 2 is illustrated in Figure 1, where the first (homogeneous) term in this equation corresponds to the left picture while the second (splitting) term corresponds to the right picture. Q_s in the splitting term is the scale where the parton splitting occurs. Thus, the two partons originate either from a nucleon or from the perturbative parton splitting. The ladder diagrams represent the evolution functions, $\tilde{E}_{ab}(n, Q, Q_0)$, which are the DGLAP parton in parton distribution functions, obeying the following integral equation¹

$$\tilde{E}_{ab}(n, Q, Q_0) = T_a(Q, Q_0) \delta_{ab} + \int_{Q_0^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} T_a(Q, k_{\perp}) \sum_{a'} \tilde{P}_{aa'}(n, k_{\perp}) \tilde{E}_{a'b}(n, k_{\perp}, Q_0), \quad (4)$$

where $\tilde{P}_{aa'}(n, k_{\perp})$ is the Mellin transformed splitting function with k_{\perp} -dependence in the strong coupling constant, and

$$T_a(Q, k_{\perp}) = \exp \left\{ - \int_{k_{\perp}^2}^{Q^2} \frac{dp_{\perp}^2}{p_{\perp}^2} \sum_{a'} \int_0^1 dz z P_{a'a}(z, k_{\perp}) \right\} \quad (5)$$

is the Sudakov formfactor which sums virtual corrections.

UNINTEGRATED DOUBLE PARTON DISTRIBUTIONS

We have already introduced all the elements to unfold the transverse momentum dependence from Equation 2. In the DGLAP scheme, the k_{\perp} in Equation 4 is the transverse momentum of the t -channel (exchanged) parton. Thus, after substituting (4) into (2), we unfold the k_{\perp} -dependence from the DPDFs as integrands of the k_{\perp} integrations.

For the first, homogeneous term in Equation 2, we find after such a substitution

$$\begin{aligned} \tilde{D}_{a_1 a_2}^{(h)}(n_1, n_2, Q_1, Q_2) &= T_{a_1}(Q_1, Q_0) T_{a_2}(Q_2, Q_0) \tilde{D}_{a_1 a_2}(n_1, n_2, Q_0, Q_0) \\ &+ \int_{Q_0^2}^{Q_2^2} \frac{dk_{2\perp}^2}{k_{2\perp}^2} \left\{ T_{a_1}(Q_1, Q_0) T_{a_2}(Q_2, k_{2\perp}) \sum_b \tilde{P}_{a_2 b}(n_2, k_{2\perp}) \tilde{D}_{a_1 b}^{(h)}(n_1, n_2, Q_0, k_{2\perp}) \right\} \\ &+ \int_{Q_0^2}^{Q_1^2} \frac{dk_{1\perp}^2}{k_{1\perp}^2} \left\{ T_{a_1}(Q_1, k_{1\perp}) T_{a_2}(Q_2, Q_0) \sum_b \tilde{P}_{a_1 b}(n_1, k_{1\perp}) \tilde{D}_{ba_2}^{(h)}(n_1, n_2, k_{1\perp}, Q_0) \right\} \\ &+ \int_{Q_0^2}^{Q_1^2} \frac{dk_{1\perp}^2}{k_{1\perp}^2} \int_{Q_0^2}^{Q_2^2} \frac{dk_{2\perp}^2}{k_{2\perp}^2} \left\{ T_{a_1}(Q_1, k_{1\perp}) T_{a_2}(Q_2, k_{2\perp}) \sum_{b,c} \tilde{P}_{a_1 b}(n_1, k_{1\perp}) \tilde{P}_{a_2 c}(n_2, k_{2\perp}) \tilde{D}_{bc}^{(h)}(n_1, n_2, k_{1\perp}, k_{2\perp}) \right\}, \end{aligned} \quad (6)$$

where the functions $\tilde{D}^{(h)}$ on the right hand side are the homogeneous DPDFs evolved from initial conditions like on the left picture in Figure 1, e.g

$$\tilde{D}_{bc}^{(h)}(n_1, n_2, k_{1\perp}, k_{2\perp}) = \sum_{a', a''} \tilde{E}_{ba'}(n_1, k_{1\perp}, Q_0) \tilde{E}_{ca''}(n_2, k_{2\perp}, Q_0) \tilde{D}_{a' a''}(n_1, n_2, Q_0, Q_0). \quad (7)$$

¹This equation can be readily recast into the differential form of the DGLAP equation with the initial condition $\tilde{E}_{ab}(n, Q_0, Q_0) = \delta_{ab}$.

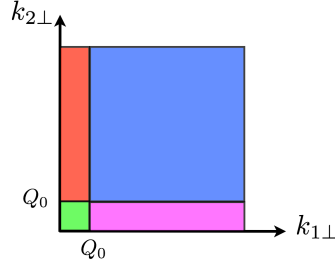


FIGURE 2. Regions of transverse momenta for the homogeneous part of the UDPDFs, defined through Equation 6.

The integrands in the curly brackets in Equation 6 are the unintegrated DPDFs, $\tilde{f}_{a_1 a_2}^{(h)}$, which are defined in three regions of the transverse momentum plane, $(k_{1\perp}, k_{2\perp})$, shown in Figure 2 as the red, pink and blue rectangles. The first two expressions (from the top) in Equation 6 have one of the two transverse momenta equal to the initial scale Q_0 . It means that this transverse momentum is integrated up to Q_0 and is not present among the arguments of the defined function. The effect of such an integration is hidden in the integrated DPDFs, $\tilde{D}^{(h)}$, taken at Q_0 for one of the two scales. Such UDPDFs correspond to the red and pink regions in Figure 2. The UDPDF in the blue region is defined by the third term in the curly brackets in Equation 6, which after transforming back to the x -space reads

$$f_{a_1 a_2}^{(h)}(x_1, x_2, k_{1\perp}, k_{2\perp}, Q_1, Q_2) = T_{a_1}(Q_1, k_{1\perp}) T_{a_2}(Q_2, k_{2\perp}) \times \sum_{b,c} \int_{\frac{x_1}{1-x_2}}^{1-\Delta_1} \frac{dz_1}{z_1} \int_{\frac{x_2}{1-x_1/z_1}}^{1-\Delta_2} \frac{dz_2}{z_2} P_{a_1 b}(z_1, k_{1\perp}) P_{a_2 c}(z_2, k_{2\perp}) D_{bc}^{(h)}\left(\frac{x_1}{z_1}, \frac{x_2}{z_2}, k_{1\perp}, k_{2\perp}\right). \quad (8)$$

The upper limits in the integrals above are shifted from 1 by $\Delta_i = k_{i\perp}/Q_i$ for $i = 1, 2$ to regularize the divergence of the flavor diagonal splitting functions at $z = 1$. The same procedure is applied to the Sudakov formfactor (5). A closer inspection of Equation 8 shows that the longitudinal and transverse momenta are correlated by the relation

$$\frac{x_1}{1-\Delta_1} + \frac{x_2}{1-\Delta_2} \leq 1, \quad (9)$$

which is a stronger condition than that for the DPDFs, $x_1 + x_2 \leq 1$. Finally, the green region in Figure 2 corresponds to the first term in Equation 6 in which both transverse momenta are integrated up to the scale Q_0 . Thus, there are no UDPDFs in this region.

In principle, all the regions of transverse momenta need to be included for any configuration of the external hard scales Q_1 and Q_2 . It is clear though, that some regions will be subdominant depending on the scales, due to the suppression originating from the Sudakov formfactors. For example, the first term in Equation 6 is going to be very small whenever any of the scales is much larger than Q_0 .

DISCUSSION OF THE SPLITTING CONTRIBUTION

The discussion of the splitting contribution is more involved since in principle there are two potential sources of transverse momenta dependence in this case, from the splitting vertex itself and from the evolution above the splitting vertex. In the latter case, we can apply the method from the previous section to the second term in Equation 2. For example, in the blue region in Figure 2, i.e. for $k_{1\perp}, k_{2\perp} \geq Q_0$, we find the following unintegrated DPDFs in the x -space

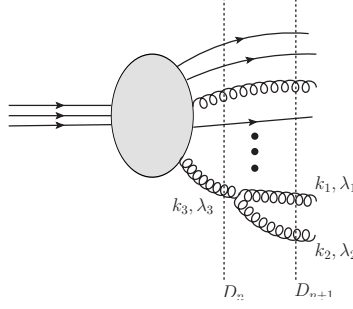


FIGURE 3. Splitting contribution to the proton wave function in the light-front framework.

from the splitting mechanism [10]

$$f_{a_1 a_2}^{(sp)}(x_1, x_2, k_{1\perp}, k_{2\perp}, Q_1, Q_2) = T_{a_1}(Q_1, k_{1\perp}) T_{a_2}(Q_2, k_{2\perp}) \int_{\frac{x_1}{1-x_2}}^{1-\Delta_1} \frac{dz_1}{z_1} \int_{\frac{x_2}{1-x_1/z_1}}^{1-\Delta_2} \frac{dz_2}{z_2} \sum_{b,c} P_{a_1 b}(z_1, k_{1\perp}) P_{a_2 c}(z_2, k_{2\perp}) \times \int_{Q_0^2}^{Q_s^2} \frac{dQ_s^2}{Q_s^2} \theta(k_{1\perp}^2 - Q_s^2) \theta(k_{2\perp}^2 - Q_s^2) \mathcal{D}_{bc}^{(sp)}\left(\frac{x_1}{z_1}, \frac{x_2}{z_2}, k_{1\perp}, k_{2\perp}, Q_s\right), \quad (10)$$

where the integrated distribution $\mathcal{D}_{bc}^{(sp)}$ on the right hand side is obtained from the two ladder evolution, i.e. in the Mellin moment space

$$\tilde{\mathcal{D}}_{bc}^{(sp)}(n_1, n_2, k_{1\perp}, k_{2\perp}, Q_s) = \sum_{a', a''} \tilde{E}_{ba'}(n_1, k_{1\perp}, Q_s) \tilde{E}_{ca''}(n_2, k_{2\perp}, Q_s) \tilde{\mathcal{D}}_{a'a''}^{(sp)}(n_1, n_2, Q_s). \quad (11)$$

By the comparison with Equation 7 we see that the "initial condition" for this evolution is given by the effective distribution (3) which contains the single PDFs taken at the splitting scale Q_s . The analysis of the two remaining contributions, in which one of the two momenta is integrated out, reveals that they cannot be treated on the same footing as those in the homogeneous case. The reason is the lack of a clear cut division between the perturbative and non-perturbative regions since the integration over the transverse momentum extends up to Q_s which can be much bigger than Q_0 [10]. Thus only formula (10), valid for $k_{1\perp}, k_{2\perp} \geq Q_0$, is acceptable in the discussed case.

For the discussion of the transverse momentum dependence coming directly from the perturbative splitting of a single parent parton into two daughter partons, we utilized the methods of the light-front perturbation theory. Applying these methods to the diagram shown in Figure 3 we were able to find the result (3) for the splitting term in the evolution equations (2) for the integrated DPDFs. Going deeper into the transverse momentum dependence of the expressions leading to this results, we found in the strong ordering approximation of transverse momenta, $k_{\perp} \simeq k_{1\perp} \simeq k_{2\perp} \gg k_{3\perp}$, the following unintegrated DPDFs form the splitting vertex

$$f_{a_1 a_2}(x_1, x_2, \mathbf{k}_{1\perp}, \mathbf{k}_{2\perp}) = \frac{\alpha_s}{2\pi} \frac{1}{x_1 + x_2} \frac{k_{1\perp}^2 k_{2\perp}^2}{k_{3\perp}^2 k_{\perp}^2} P_{a_1 a} \left(\frac{x_1}{x_2 + x_1} \right) f_a(x_1 + x_2, \mathbf{k}_{3\perp}), \quad (12)$$

where $f_a(x_1 + x_2, \mathbf{k}_{3\perp})$ is the unintegrated single PDFs. Applying the method of References [8, 9], this distribution can be given the dependence on the hard scale Q ,

$$f_a(x_1 + x_2, k_{3\perp}, Q) = T_a(Q, k_{3\perp}) \sum_{a'} \int_{x_1+x_2}^{1-\Delta} \frac{dz}{z} P_{aa'}(z, k_{3\perp}) D_{a'} \left(\frac{x_1 + x_2}{z}, k_{3\perp} \right), \quad (13)$$

where $\Delta = k_{3\perp}/Q$. Thus the distribution (12) becomes scale dependent with equal scales, $Q_1 = Q_2 = Q$,

$$f_{a_1 a_2}(x_1, x_2, k_{1\perp}, k_{2\perp}, Q, Q) = \frac{\alpha_s}{2\pi} \frac{1}{x_1 + x_2} \frac{k_{1\perp}^2 k_{2\perp}^2}{k_{3\perp}^2 k_{\perp}^2} P_{a_1 a} \left(\frac{x_1}{x_2 + x_1} \right) f_a(x_1 + x_2, k_{3\perp}, Q). \quad (14)$$

The reason for equal scales is that formula (14) only contains evolution of the unintegrated single parton density up to a scale Q and then the splitting is treated with the transverse momentum dependence. The two partons from the splitting should evolve now. However, the initial partons have nonzero transverse momenta which may be from the perturbative region, $k_{1\perp}, k_{2\perp} \geq Q_0$. Thus, we should consider QCD radiation with transverse momentum dependent splitting functions, see e.g. [13] for details. This stays, however, beyond the scope of the present analysis.

CONCLUSIONS

Following the method of Kimber, Martin and Ryskin (KMR) [8, 9], we presented main points of the construction of unintegrated double parton distribution functions which depend on parton transverse momenta, $k_{1\perp}$ and $k_{2\perp}$, in addition to their two longitudinal momentum fractions, x_1 and x_2 , and two factorization scales, Q_1 and Q_2 . We discussed two contributions to the unintegrated DPDFs, corresponding to the possibility that the two partons originate either from the proton or from the splitting of a single parton. In the first case the main formula is given by Equation 8. It corresponds to the fully perturbative domain of transverse momenta, $k_{1\perp}, k_{2\perp} \geq Q_0$. The formulae in the half-perturbative domains are presented in Reference [10].

In the perturbative case with parton splitting, we discussed two cases, the unfolding of the transverse momentum dependence from the last step in the DGLAP evolution of two partons, and the case where the transverse momenta are generated directly from the single parton splitting into two partons. In the first case, only formula (10) with perturbative transverse momenta makes sense. In the second case, we propose formula (14) which includes transverse momentum dependence generated from the perturbative splitting of one parton into two daughter partons. In that case, the KMR prescription is applied to the single PDF, in order to introduce the transverse momentum dependence, and then the splitting is treated by including the transverse momentum dependence. We kept the derivation in the strong ordering approximation to be consistent with the rest of the framework. The discussion of more subtle aspects of the transverse momentum dependence of UDPDFs, like the dependence on an additional transverse momentum \mathbf{r}_\perp or the transverse momentum dependence of the evolution after the parton splitting, as well as a numerical analysis, have to be postponed to future publications.

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